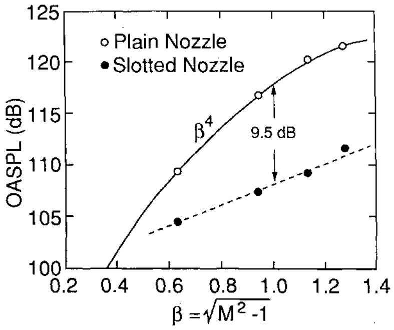

Fig. 4 The variation of the OASPL with parameter β; measurements are made at 90 deg to the jet axis.

cal structures commonly observed in axisymmetric jets can be seen from these photographs and are indicated by the arrows. For downstream locations of $X/D > 5$, these large structures are not destroyed by the presence of the slots.

A typical variation of the pitot presure P_c along the centerline of the jet for a pressure ratio of 4.4 is shown in Fig. 2. The absolute values of P_c are divided by the stagnation pressure P_0 and plotted against the nondimensionalized downstream distance. These results cannot be converted directly into velocity or Mach number because of the unknown entropy increase that has occurred. In a steady supersonic flow with a single normal shock wave ahead of the pitot tube, a large pitot pressure corresponds to low Mach number, and vice versa. For the case of the plain nozzle, near the nozzle exit, a sharp drop in the pitot pressure followed by a rise is observed, which signifies the presence of the normal shock wave commonly known as the Mach disc. For $X/D > 8$, the total pressure decays monotonically and has a variation similar to a subsonic jet. The variation of the total pressure along the centerline of the jet exiting from a slotted nozzle suggests that the shock cell structure is significantly weakened by the presence of the slots. Similar conclusions are reached from observations of total pressure profiles taken across the jet at different downstream locations.[5]

Figure 3 shows the distribution of the pitot pressure across the jet in the central plane at $X/D = 10$ and for both plain and slotted nozzles. This downstream location is well beyond the shock cell region of the jet. The pressure is normalized with respect to the centerline pressure P_c, and the distance r is normalized with the exit radius of the nozzle. It appears that both profiles have similar distribution. These and other measurements (see Ref. 5) taken at different downstream locations suggest that for $X/D \geq 10$ the influence of the fingers on the overall structure of the jet seems to be negligible. However, because of the difference in the magnitudes of the centerline pressures (see Fig. 2), the overall thrust obtained by integrating the pressure across the jet indicates that some thrust loss may occur due to the presence of the fingers or slots.

The effect of the slotting on the far-field noise can be seen in Fig. 4, which is a plot of overall sound pressure level (OASPL) against the parameter $\beta(=\sqrt{M^2 - 1})$, where M is the exit Mach number assuming an ideally expanded jet near the nozzle exit. These measurements are made at $\theta = 90$ deg and at 100 diam from the jet axis. In the case of the jet exiting from the plain nozzle, the measured levels are directly proportional to the fourth power of β, an observation consistent with other investigations. As pointed out by Harper-Bourne and Fisher,[6] the β^4 dependence indicates the dominant contribution of the shock-associated noise to the OASPL. However, measurements made with the slotted nozzle do not follow this trend, and a significant reduction in the OASPL is observed as shown in the figure. Based on the flow visualization pictures and the previous observation, it may be suggested that the reduction in the magnitude of the OASPL is due primarily to the absence of the screech tones and significantly reduced shock-associated broadband noise.

Conclusions

From these preliminary experiments, it appears that the addition of fingers or slots to a converging axisymmetric nozzle contributes to a significant noise reduction. The slots perform as silencers because they weaken the shock cell structure near the nozzle exit and thereby reduce the shock-associated noise. These slots also help to enhance the mixing close to the nozzle exit ($X/D < 10$). The present study is preliminary in nature and all implications of the results presented are not yet fully understood. Further detailed investigations are clearly needed to clarify the importance of the flow details at the nozzle exit and the flow structure close to it.

Acknowledgment

This work was carried out at Stanford University during 1978 in the Department of Aeronautics and Astronautics.

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Nonlinear Analysis of an Adhesive-Bonded Joint Under Generalized In-Plane Loading

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Nomenclature

A^i_{mn}, A^o_{mn} = adherend extension stiffnesses (m,n = 1 or 2)
a,b = bond dimensions
G_c = initial slope of the adhesive stress strain curve
G_s = adhesive secant modulus
S_{12}, S_{21} = uniformly distributed resultant in-plane shear loads

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T_1, T_2 = uniformly distributed resultant axial loads
γ_{eff} = effective adhesive shear strain
γ_x, γ_y = adhesive shear strain components
η = bond line thickness
λ_{11}, λ_{22} = compliance parameters
μ = adhesive secant modulus parameter
τ_{eff} = effective adhesive shear stress
τ_p = adhesive proportional limit stress
τ_x, τ_y = adhesive shear stress components

Introduction

ADHESIVE-BONDED joints have widespread applications in aerospace structures due to their minimal weight and efficient load transfer characteristics. The analysis of bonded joints under simple loading conditions has received considerable attention, and design methods that account for adhesive nonlinearity are available.[1,2] The development of bonded wing root joints for future aircraft will require analysis and design methods that account for adhesive nonlinearity under complex loading conditions. This Note presents a nonlinear stress analysis method for adhesive-bonded joints under generalized in-plane loading. The method is developed for a double-lap joint but can be readily extended to other bonded joint configurations.

Analysis

A schematic of the double-lap joint under generalized in-plane loading is shown in Fig. 1. In general, the inner and outer adherends are composed of different orthotropic materials. Thermoelastic effects resulting from the bonding of dissimilar materials can be included in the analysis but are not discussed here.

The adherends are modeled as thin elastic plates under membrane loading. The adhesive is assumed to be characterized by a single-valued stress strain relation of the form $\tau = f(\gamma)$ for simple shear. Adhesive peel stresses are not considered in the analysis. In structural joints, these peel stresses are usually relieved by tapering the edges of the outer adherends and thickening the bond line near the edge of the overlap, as discussed by Hart-Smith.[1,3]

For generalized in-plane loading of the joint, there are two shear stress and shear strain components in the adhesive bond. Adhesive deformation under these conditions is assumed to be governed by effective stress and strain measures that obey the simple shear stress strain relation of the adhesive. From continuum plasticity theory,[4] the appropriate measures for the present problem are the adhesive principal stress and strain. Therefore

$$\tau_{eff} = (\tau_x^2 + \tau_y^2)^{1/2} \tag{1a}$$

$$\gamma_{eff} = (\gamma_x^2 + \gamma_y^2)^{1/2} \tag{1b}$$

The stress and strain components at a point in the adhesive are given by

$$\tau_x = G_s \gamma_x \tag{2a}$$

$$\tau_y = G_s \gamma_y \tag{2b}$$

where $G_s = f(\gamma_{eff})/\gamma_{eff}$.

The differential equations governing one-dimensional deformation of the double-lap joint are given by Hart-Smith.[1] Analogous equations for two-dimensional deformation can be obtained by assuming that the adherends deform as a network of orthogonal one-dimensional strips. For biaxial loading, the strips deform by simple extension and the governing differential equations are

$$\frac{d^2\gamma_x}{dx^2} - \mu\lambda_{11}^2\gamma_x = 0 \tag{3}$$

$$\frac{d^2\gamma_y}{dy^2} - \mu\lambda_{22}^2\gamma_y = 0 \tag{4}$$

where

$$\lambda_{mm}^2 = \frac{G_c}{\eta}\left(\frac{1}{A_{mm}^o} + \frac{2}{A_{mm}^i}\right) \tag{5}$$

for mm = 11 or 22. Boundary conditions relating the derivatives of γ_x, γ_y to the axial loads T_1, T_2 can be derived by considering the extension of the strips. For in-plane shear loading, a similar boundary value problem can be obtained by assuming that the strips deform in simple shear under the shear loads S_{12}, S_{21}.

The differential equations of γ_x, γ_y are nonlinear by virtue of μ, which is a nonlinear function of γ_{eff}. The nonlinear boundary value problems can be solved by the following iterative procedure:

1) Obtain the solution of γ_x, γ_y in a linearly deforming adhesive where $\mu = G_s/G_c = 1$.

2) Use the solution from step 1 to calculate μ at specified points in the adhesive bond.

3) Solve the boundary value problems for γ_x, γ_y using the updated values of μ.

4) Calculate new values of μ from the solution in step 3.

5) Repeat steps 3 and 4 until a convergent value of γ_{eff} is obtained at the specified points in the adhesive bond.

Adhesive stresses can be calculated from the convergent adhesive strain solution by using Eqs. (2).

The one-dimensional strip approximation used in the analysis was assessed by comparison with finite element results for bonded doublers under biaxial loading. The approximate method was found to provide a conservative estimate of the adhesive stresses and strains. The loss of accuracy is relatively small if the Poisson ratios of the adherends are < 0.3.

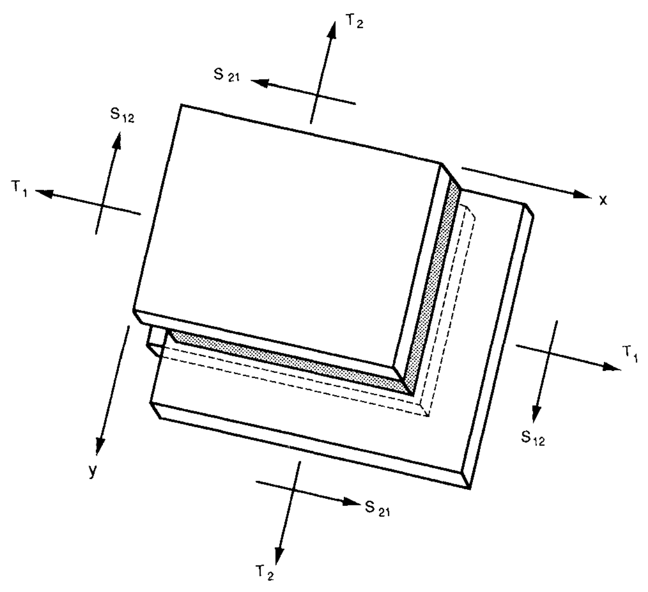

Fig. 1 Schematic of adhesive-bonded double-lap joint under generalized in-plane loading.

Results

The analysis described in the previous section was used to calculate the adhesive shear strength of the double-lap joint for various loading conditions. The joint loads were adjusted iteratively until the maximum adhesive strain equaled the ad-

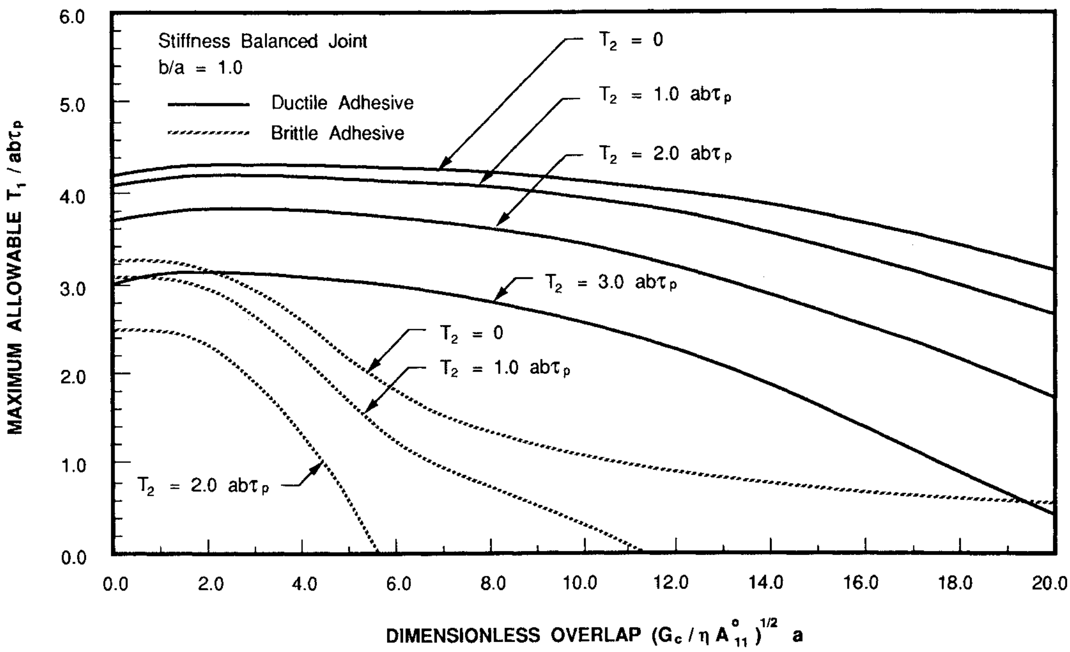

Fig. 2 Allowable average bond stress due to tension loading.

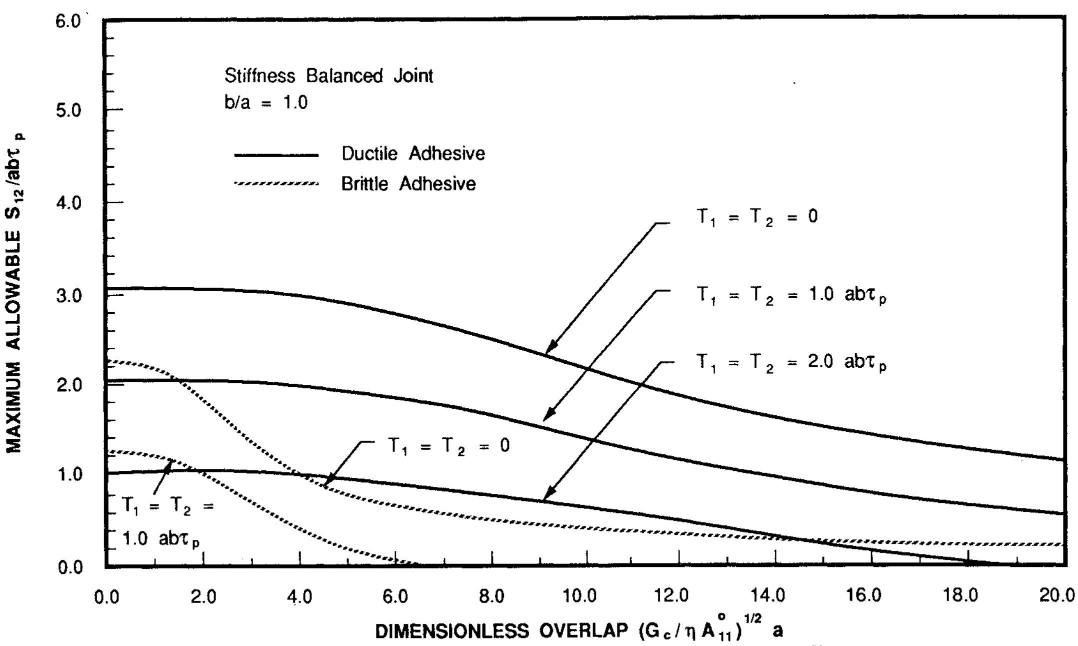

Fig. 3 Allowable average bond stress due to in-plane shear loading.

hesive strain capacity. Characteristic ductile and brittle adhesive stress strain relations were used in the calculations.

Figure 2 shows the variation of adhesive joint strength with overlap size for biaxial load conditions. These results show that the maximum allowable average bond stress associated with the load T_1 decreases with increasing biaxial loading and overlap size. This trend suggests that, for a given loading condition, there is a critical bond size at which the joint develops its maximum strength. Further increases in bond size beyond this critical value will not increase the strength of the joint. Similar results are shown in Fig. 3, where the maximum allowable average bond stress associated with the shear load S_{12} is given for different levels of fully biaxial loading. The results of Figs. 2 and 3 show that the brittle adhesive sustains a lower average stress relative to its proportional limit τ_p than does the ductile adhesive.

Comparison of Figs. 2 and 3 reveals that the load capacity of the joint for uniaxial tension loading is substantially higher than the corresponding load capacity for in-plane shear loading. This effect is related to the difference between the extensional and shear stiffnesses of the adherends. In the usual case where the adherend extensional stiffness is greater than the adherend shear stiffness, a tension load transmits less strain to the adhesive than does a shear load of equal magnitude. Since bond failure is caused by the attainment of a critical adhesive strain, the shear load capacity of these joints is lower. For composite bonded joints, this effect can be reduced by tailoring the adherends to have greater shear stiffness.

Summary

This Note describes a nonlinear stress analysis procedure for bonded joints under generalized in-plane loading. The procedure was used to calculate the adhesive strength of bonded double-lap joints. The results of these calculations show the effects of loading condition, bond size, and adhesive material behavior on bonded joint strength.

Acknowledgments

This work was conducted as part of Northrop Corporation's independent research and development activity on strength and life assurance of composite structures. The author is grateful to Ravi B. Deo and Dinesh S. Joshi for helpful suggestions.

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Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Direct Component Modal Synthesis Technique for System Dynamic Analysis"

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THE authors of Ref. 1 have presented an excellent method which has a number of advantages over conventional component modal synthesis techniques. By eliminating the usual displacement coordinates in favor of the joint or generalized, forces between substructures, the size of the governing matrix equations is substantially reduced. Indeed, the order of the matrix is equal to the number of compatibility, or constraint, connection, and joint, conditions between substructures. The authors' examples and conclusions further support the advantages of the method.

The present writer is particularly pleased to see the publication of this paper as he developed essentially the same method some years ago, and described it in a series of publications. In the present writer's approach, the Lagrange Multiplier method is used to incorporate the constraint or compatibility conditions between substructures and the forces of constraint or compability are simply the Lagrange Multipliers themselves.[2-9] This method has also been used to incorporate the effects of nonconservative forces including damping,[5] as well as nonlinearities.[6-8]

It has also been shown how the method may be used to add or subtract a substructure without a total reanalysis of the system.[9]

Now that the method has been rediscovered, it will hopefully be more widely used.

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Reply by the Author to Earl H. Dowell

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We appreciate the comments from Prof. E. H. Dowell. Dowell presented a method, as discussed in his references, which is a variation of a Rayleigh Ritz approach to determine the system eigen frequency. The Lagrange multiplier lambda vector in his method corresponds to the modal force vector in the modal force method. Contrary to Dowell's variational approach, we formulate the solution equation by physical coordinates. Futhermore, after the modal force vector has been obtained, the modal force method provides the mode shape of the entire system in physical coordinates directly without any inversion of matrix.

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